Advanced Risk and Performance Measures

The Rock Creek Group
May 11, 2009

Hedge funds tend to use investment tools with non-linear payoff and as such their return distribution are non-Gaussian. For this reason traditional risk measures are inadequate, as indeed are risk adjusted performance measures such as Sharpe ratios that may be more easily manipulated. Moreover, traditional risk measures tend to be highly correlated with one another, and as such using a number of these measures does not provide additional insight into the riskiness of portfolios. This note studies alternative risk measures that tend to be less correlated with traditional risk measures and which are more appropriate for non-Gaussian distributions. The note compares traditional and alternative risk measures for group of 848 hedge funds using monthly data between March 2004 and February 2009. Only funds with at least 36 months of data during this period were included in the analysis.

I. Traditional Risk Measure

We calculate four widely used traditional risk measures on our database of hedge fund returns. These include:

- volatility (σ),
- worst return (r_{min}),
- value at risk at 1% level ($VaR_{1\%}$),
- maximum drawdown (MaxDD).

Table 1 provides the correlations between these four measures.

	σ	r_{min}	$VaR_{1\%}$	MaxDD
σ	1	-0.88	-0.99	-0.81
r_{min}	-0.88	1	0.91	0.87
$VaR_{1\%}$	-0.99	0.91	1	0.87
MaxDD	-0.81	0.87	0.87	1

Table 1: The correlations between traditional risk measures

It is clear these four risk measures are strongly correlated with one another, especially value at risk and the volatility. The strong correlations demonstrate that using all four risk measures to determine the riskiness of a portfolio is largely redundant. If the volatility is used as the risk measure, then there is very little additional information to be gleaned by using value at risk. While there is some marginal information in maximum drawdown, it is limited. In short, using more risk measures does not necessarily equate to more accurate measurement of risk. More likely, they are just different ways of displaying the same information. A failure to recognize this fact could lead to a false sense of comfort in estimating the risks of an investment portfolio.

II. Cornish Fisher Value at Risk

One of the alternative risk measures that is commonly used in the literature is Cornish Fisher Value at risk ($VaR_{p,cf}$),While the traditional VAR only uses the second moment of a return distribution, i.e. volatility, the $VaR_{p,cf}$ uses the third and fourth moments of a distribution namely skew and kurtosis. However, $VaR_{p,cf}$ is a good approximation of VAR if and only if :

- The return distribution is reasonably close to being Gaussian, In other words, the skew and kurtosis are close to 0.
- The confidence level, at which VaR is calculated, is not very low.

In our data sample, about 48% of the hedge funds have skew less than -1, and as such the first requirement for using $VaR_{p.cf}$ is not met. While most traditional VAR measures use a 5% confidence level, we believe that measuring risk at this level is inadequate. This is because with a $VaR_{5\%}$ level calculated on monthly returns the probability of a loss larger than that level over a year is 46%. This is not a rare event and therefore not acceptable as a risk measure. If we wish to make the VAR level more meaningful we should calculate VaR at the 1% confidence level; in that case the second requirement for using $VaR_{p.cf}$ is not satisfied. When both requirements are not satisfied, $VaR_{p,cf}$ is not a good risk measure. A quick analysis confirms this point. The standard VaR_p and $VaR_{p,cf}$ of a portfolio are defined as:

$$VaR_p = \bar{r} + \sigma z_p \quad (1)$$

$$VaR_{p,cf} = \bar{r} + \sigma z_{p,cf} \quad (2)$$

where $\bar{r}$ is the mean return, and z_p is the quantile function of the Gaussian distribution with a standard deviation of σ, and

$$z_{p,cf} = z_p + \frac{z_p^2 - 1}{6} S + \frac{z_p^3 - 3z_p}{24} K + \frac{5z_p - 2z_p^3}{36} S^2 \quad (3)$$

is the Cornish Fisher quantile function of the distribution with skew S and kurtosis K[1]. We would expect $VaR_{p,cf}$ to be smaller than VaR_p for negative values of S. For this to hold,

- Both z_p and $z_{p,cf}$ should be negative
- Both z_p and $z_{p,cf}$ should decrease as p decreases.
- $z_{p,cf}$ should decrease as S decreases.

These conditions are met for z_p, but do not always hold true for $z_{p,cf}$. It is clear that the last term in Eq. 3 will be positive

[1] In this note, parameter K is the excess kurtosis, and $K = 0$ for the Gaussian distribution.

when both S and z_p are large and negative. Fig. 1 shows $z_{p,cf}$ for different values of S and p but with $K = 0$. From Fig. 1 it is clear that;

1. $z_{p,cf}$ does not monotonically decrease as p decreases.
2. $z_{p,cf}$ does not always decrease as S decreases. It increases as S deceases for small values of p.

In fact $z_{p,cf}$ stops decreasing and turns upwards when $p \leq 1\%$ and $S \leq -1$. Given that 48% of the hedge funds in our data sample have $S < -1$ we do not think Cornish Fisher value at risk is a good risk measure for hedge funds.[2]

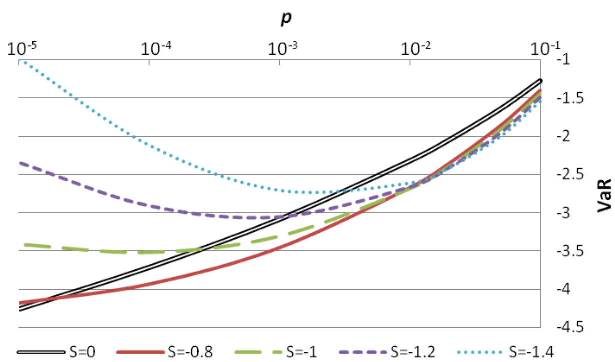

Figure 1: The VaR with different skewness levels. The horizontal axis is the probability $\boldsymbol{p}$ at which the VaR is calculated. The black double line is the normal $\boldsymbol{VaR_p}$. The red solid line is $\boldsymbol{VaR_{p,cf}(S = -0.8)}$, the Cornish Fisher VaR with skewness at -0.8. The long dashed green line is $\boldsymbol{VaR_{p,cf}(S = -1)}$. The purple dashed line is $\boldsymbol{VaR_{p,cf}(S = -1.2)}$. And the dotted line is $\boldsymbol{VaR_{p,cf}(S = -1.4)}$.

III. Extreme Value Theory

The traditional risk measures do not explicitly take into account non-Gaussian tails. And this leads to an underestimation of the risk under extreme market conditions. As discussed above, the commonly used Cornish-Fisher $VaR_{p,cf}$ fails to address these extreme cases. Extreme value theory is an alternative to study these extreme cases. Under extreme theory, the probability distribution at the tails of a return distribution is modeled as

$$f(r) = \frac{1}{\lambda}\left(1 + \xi\frac{r-\mu}{\lambda}\right)^{-1/\xi-1} \tag{4}$$

where r is the return, μ is the location parameter, the ξ is shape parameter, and λ is scale parameter. These parameters are determined from a deterministic fit to historic data. Given the distribution, the $VaR_{p,EVT}$, is determined such that

$$p = \int_{-\infty}^{VaR_{p,EVT}} f(r)dr \tag{5}$$

where p is the chosen confidence level, and is set at 1%. The $VaR_{p,EVT}$ will better capture the probability of rare events. However, given that $VaR_{p,EVT}$ is the least amount of loss if the low probability event were to happen, its use will be an underestimation of the potential loss in case of an extreme event. We therefore use the expected shortfall $ES_{p,EVT}$ measure instead of $VaR_{p,EVT}$ to capture the magnitude of the loss, where $ES_{p,EVT}$ is defined as

$$ES_{p,EVT} = \int_{-\infty}^{VaR_{p,EVT}} rf(r)dr \tag{6}$$

The expected shortfall is also called Conditional Value at Risk and is the expected loss if the extreme event were to occur.

We calculated VaR_p and $ES_{p,EVT}$ for a sample of 206 hedge funds from our database all of which had a single month loss larger than 10% in September 2008. Using data through August 2008, $ES_{p,EVT}$ identified 124 hedge funds from this group as having the potential such a loss while VaR_p identified 52 hedge funds from this group as having the potential for such a loss. More importantly, all 52 hedge funds that were identified by VaR_p were also correctly classified by $ES_{p,EVT}$. In short, the use of $ES_{p,EVT}$ significantly increases the probability of identifying hedge funds with potentially large losses in extreme situations.

We introduce a new risk measure, the RCG risk measure, $RCG_{p,EVT}$, which is also based off extreme value theory. This measure is the difference between $ES_{p,EVT}$ and the risk related to the Gaussian tail of the return distribution of the manager. In other words, it measures the unexpected risk in extreme cases. Table 2 shows the correlation between this measure and the more traditional risk measures.

	$\boldsymbol{\sigma}$	$\boldsymbol{r_{min}}$	**MaxDD**	$\boldsymbol{RCG_{1\%,EVT}}$
$\boldsymbol{\sigma}$	1	-0.88	-0.81	0.05
$\boldsymbol{r_{min}}$	-0.88	1	0.87	-0.38
MaxDD	-0.81	0.87	1	-0.28
$\boldsymbol{RCG_{1\%,EVT}}$	0.05	-0.38	-0.28	1

Table 2: The correlations among some risk measures. The RCG risk measure has very small correlation with other measures.

$RCG_{p,EVT}$ has low correlations with volatility and other traditional measures. We therefore believe that $RCG_{p,EVT}$ adds additional information to our risk estimation process. It is an alternative risk measure that is distinct from traditional risk measures.

[2] In principle, including higher order moments can help. However, there is estimation uncertainty due to the sample size. The estimation uncertainty is about 0.3 for the skewness, and is about 0.54 for the kurtosis. The estimation uncertainties for 5th and 6th order normalized moments are about 2.8 and 7.3 respectively. To reduce them to 0.5, the sample size has to be about 2400 and 24000, respectively. This is not possible for monthly return data.

IV. Bias Ratio

The bias ratio (Riskdata 2006) is a measure that seeks to measure the bias inherent in valuing illiquid assets. The bias to report slightly positive rather than slightly negative returns forms the basis for this measure. In short, portfolios with assets whose values may be valued with some discretion generally have distorted return distributions around zero. The bias ratio is defined as

$$BR = \frac{counts(r_t | r_t \in [0, \sigma])}{1 + counts(r_t | r_t \in [-\sigma, 0])} \qquad (7)$$

where r_t is the return and σ is the volatility. It is designed to detect the abnormal distortion of return distribution shown in Fig. 2.

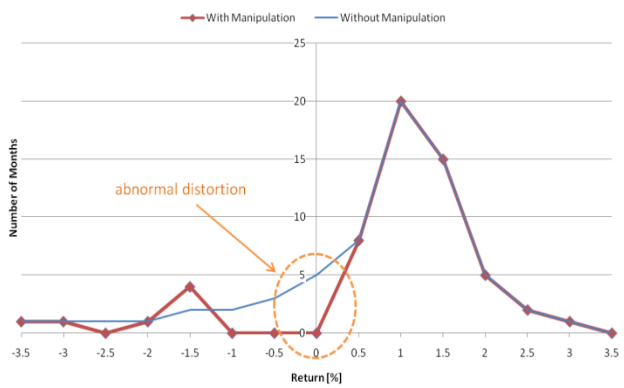

Figure 2: Definition of the Bias Ratio. The blue line is the return distribution without biased asset valuation. The red line is the return distribution with biased asset valuation.

The bias ratio positively correlates with the liquidity of the assets managed by the hedge funds, i.e. the more illiquid the underlying securities the larger the bias ratio. Calculating the ratio for return streams assists in detecting the existence of biased valuation of illiquid assets. In general, hedge funds with largely liquid assets that are exchange traded with independent valuations have smaller bias ratios between 1 and 2. A large bias ratio for such liquid funds would suggest a fraud. Funds related to the Madoff's fraud, had biased ratios of 7.5 to 8.0, even though the underlying securities were from the S&P100 stocks.

V. Performance Measures

There are many performance measures which account for both risk and return in their calculations. The most widely used performance measure is the Sharpe ratio. This uses volatility as the risk measure, which may be an inadequate measure. More importantly, the Sharpe ratio has been subject to manipulation in many ways usually involving use of instruments with non linear payoffs like puts and calls. The use of these instruments tends to convert a Gaussian distribution into a bi-modal distribution that could lead to an underestimation of volatility, and in turn higher Sharpe ratios. Usually, to overcome these problems, a multi-valued performance measure is used as they provide additional information and are harder to manipulate. However, a multi-valued performance measure makes it harder to rank performance of different portfolios, and in turn makes it harder for investors to differentiate managers who are performing well from those who are not. A single-valued performance measure (Goetzmann, et al. 2007) that does not suffer from these drawbacks may be defined as

$$\Theta = \frac{1}{(1-\rho)} \ln \left[\frac{1}{T} \sum_{t=1}^{T} \left(\frac{1 + r_{ft} + r_t}{1 + r_{ft}} \right)^{1-\rho} \right] \qquad (8)$$

where r_{ft} and r_t are the risk free rate and the excess return on the fund over period t, and ρ is relative risk aversion. This measure is based off a power utility function and not easily amenable to dynamic and static manipulations (Goetzmann, et al. 2007). The relative risk aversion parameter is based off the benchmark returns.

$$\rho = \frac{\ln[E(1 + r_f + r_b)] - \ln(1 + r_f)}{Var[\ln(1 + r_f + r_b)]} \qquad (9)$$

where r_b is the return of benchmark. Fig. 3 shows the relation between the Sharpe ratio and Θ for our database of 848 hedgefunds

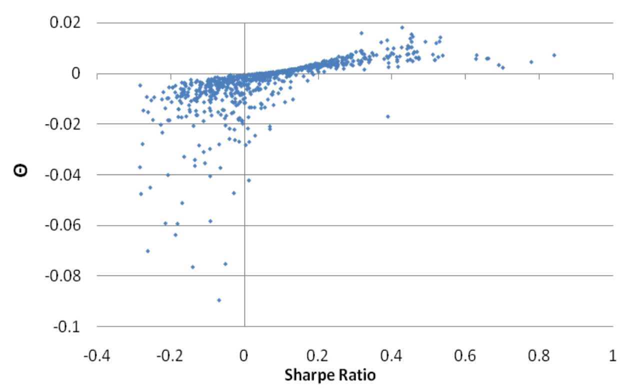

Figure 3: The Sharpe ratio and the performance measure **Θ**.

As shown in Fig. 3, the Sharpe ratio and the performance measure Θ are strongly correlated in some cases and uncorrelated in other cases. Out of 848 funds, there are 802 funds with $\Theta > -0.02$, and the correlation between Sharpe ratio and Θ is 0.79. However, if we include the 46 funds with $\Theta < -0.02$, the correlation drops to 0.33. For those 46 funds, there is no correlation between two measures. This is an indication of either an underestimation of risk or else a manipulation of Sharpe ratio.

III. CONCLUSION

In this note, we provide a brief overview of the shortcomings of some commonly used traditional risk and performance measures, and simultaneously introduce some alternative measures. Table 3 shows the correlations among all those measures. The alternative risk and performance measures introduced in this note have low correlations to traditional measures. We believe that they therefore add important new information in the estimation of risk and the measurement of performance.

	σ	r_{min}	MaxDD	$RCG_{1\%,EVT}$	Bias Ratio	Sharpe Ratio	Θ
σ	1	-0.88	-0.81	0.05	-0.14	-0.27	-0.62
r_{min}	-0.88	1	0.87	-0.38	0.05	0.44	0.71
MaxDD	-0.81	0.87	1	-0.28	0.08	0.61	0.51
$RCG_{1\%,EVT}$	0.05	-0.38	-0.28	1	0.36	-0.28	-0.15
Bias Ratio	-0.14	0.05	0.08	0.36	1	0.14	0.03
Sharpe Ratio	-0.27	0.44	0.61	-0.28	0.14	1	0.33
Θ	-0.62	0.71	0.51	-0.15	0.03	0.33	1

Table 3: The correlations between risk and performance measures

Bibliography

Goetzmann, William N., Jonathan E. Ingersoll Jr., Matthew I. Spiegel, and Ivo Welch. "Portfolio Performance Manipulation and Manipulation-proof Performance Measures." *Review of Financial Studies*, 2007: Vol. 20 No.5 1503-1546.

Riskdata. "Bias Ratio: Detecting Hedge Fund Return Smoothing." 2006.